Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematical studies
Standard level
Paper 2

Wednesday 4 November 2020 (morning)

1 hour 30 minutes

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematical studies SL formula booklet** is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- The maximum mark for this examination paper is **[90 marks]**.

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Answer **all** questions in the answer booklet provided. Please start each question on a new page. You are advised to show all working, where possible. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. Solutions found from a graphic display calculator should be supported by suitable working, for example, if graphs are used to find a solution, you should sketch these as part of your answer.

1. [Maximum mark: 19]

Don took part in a project investigating wind speed, $x\,\text{km h}^{-1}$, and the time, y minutes, to fully charge a solar powered robot.

The investigation was carried out six times. The results are recorded in the table.

Wind Speed, x, (km h^{-1})	6	10	16	24	28	30
Time, y, (minutes)	28	26	30	33	38	37

(a) **On graph paper**, draw a scatter diagram to show the results of Don's investigation. Use a scale of $1\,\text{cm}$ to represent 2 units on the x-axis, and $1\,\text{cm}$ to represent 5 units on the y-axis. [4]

(b) Calculate

(i) $\overline{x}$, the mean wind speed;

(ii) $\overline{y}$, the mean time to fully charge the robot. [2]

M is the point with coordinates $(\overline{x}, \overline{y})$.

(c) Plot and label the point M on your scatter diagram. [2]

(d) (i) Calculate r, Pearson's product–moment correlation coefficient.

(ii) Describe the correlation between the wind speed and the time to fully charge the robot. [4]

(e) (i) Write down the equation of the regression line y on x, in the form $y = mx + c$.

(ii) Draw this regression line on your scatter diagram.

(iii) Hence or otherwise estimate the charging time when the wind speed is $27\,\text{km h}^{-1}$. [6]

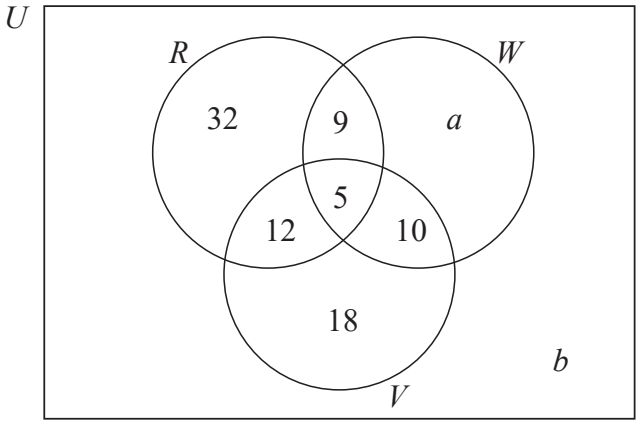
Don concluded from his investigation: "There is no causation between wind speed and the time to fully charge the robot".

(f) In the context of the question, briefly explain the meaning of "no causation". [1]

2. [Maximum mark: 12]

On a school excursion, 100 students visited an amusement park. The amusement park's main attractions are rollercoasters (R), water slides (W), and virtual reality rides (V).

The students were asked which main attractions they visited. The results are shown in the Venn diagram.



A total of 74 students visited the rollercoasters or the water slides.

(a) (i) Find the value of a.

(ii) Find the value of b. [4]

(b) Find the number of students who visited at least two types of main attraction. [2]

(c) Write down the value of $n(R \cap W)$. [1]

(d) Find the probability that a randomly selected student visited

(i) the rollercoasters;

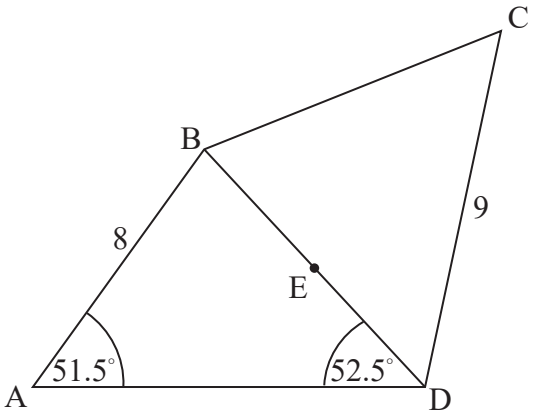
(ii) the virtual reality rides. [3]

(e) Hence determine whether the events in **parts (d)(i)** and **(d)(ii)** are independent. Justify your reasoning. [2]

3. [Maximum mark: 15]

Using geometry software, Pedro draws a quadrilateral $ABCD$. $AB = 8\,\text{cm}$ and $CD = 9\,\text{cm}$. Angle $BAD = 51.5^\circ$ and angle $ADB = 52.5^\circ$. This information is shown in the diagram.

diagram not to scale



(a) Calculate the length of BD. [3]

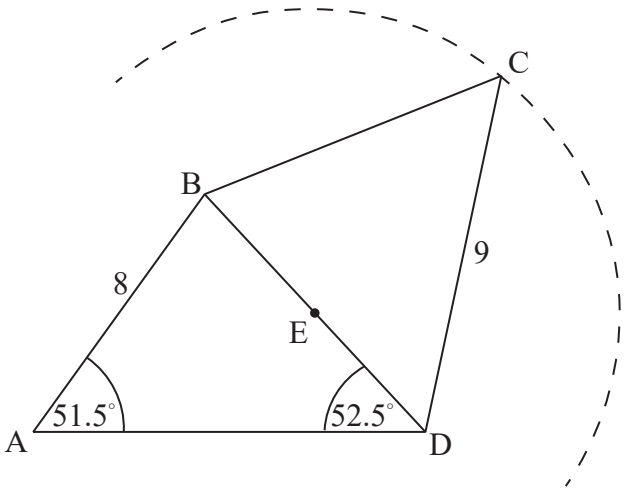
$CE = 7\,\text{cm}$, where point E is the midpoint of BD.

(b) Show that angle $EDC = 48.0^\circ$, correct to three significant figures. [4]

(c) Calculate the area of triangle BDC. [3]

Pedro draws a circle, with centre at point E, passing through point C. Part of the circle is shown in the diagram.

diagram not to scale

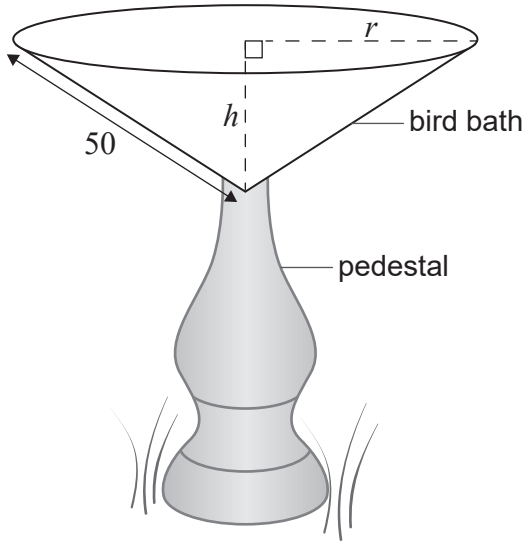


(d) Show that point A lies outside this circle. Justify your reasoning. [5]

4. [Maximum mark: 11]

Hyungmin designs a concrete bird bath. The bird bath is supported by a pedestal. This is shown in the diagram.

diagram not to scale



The interior of the bird bath is in the shape of a cone with radius r, height h and a constant slant height of $50\,\text{cm}$.

(a) Write down an equation in r and h that shows this information. [1]

Let V be the volume of the bird bath.

(b) Show that $V = \frac{2500\pi h}{3} - \frac{\pi h^3}{3}$. [1]

(c) Find $\frac{\mathrm{d}V}{\mathrm{d}h}$. [2]

Hyungmin wants the bird bath to have maximum volume.

(d) Using your answer to **part (c)**, find the value of h for which V is a maximum. [2]

(e) Find the maximum volume of the bird bath. [2]

To prevent leaks, a sealant is applied to the interior surface of the bird bath.

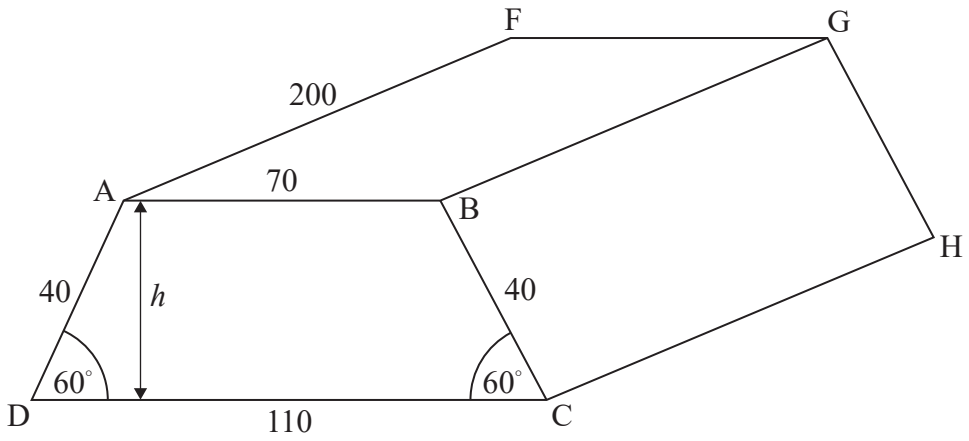
(f) Find the surface area to be covered by the sealant, given that the bird bath has maximum volume. [3]

5. [Maximum mark: 16]

A large underground tank is constructed at Mills Airport to store fuel.
The tank is in the shape of an isosceles trapezoidal prism, $\mathrm{ABCDEFGH}$.

$\mathrm{AB} = 70\,\mathrm{m}$, $\mathrm{AF} = 200\,\mathrm{m}$, $\mathrm{AD} = 40\,\mathrm{m}$, $\mathrm{BC} = 40\,\mathrm{m}$ and $\mathrm{CD} = 110\,\mathrm{m}$. Angle $\mathrm{ADC} = 60^\circ$ and angle $\mathrm{BCD} = 60^\circ$. The tank is illustrated below.

diagram not to scale



(a) Find h, the height of the tank. [2]

(b) Show that the volume of the tank is $624\,000\,\mathrm{m}^3$, correct to three significant figures. [3]

Once construction was complete, a fuel pump was used to pump fuel **into** the empty tank. The amount of fuel pumped into the tank by this pump **each hour** decreases as an arithmetic sequence with terms $u_1, u_2, u_3, \ldots, u_n$.

Part of this sequence is shown in the table.

Hour (n)	1st	2nd	3rd	…
Amount of fuel pumped into the tank in each hour, in m^3 (u_n)	45 000	43 200	41 400	…

(c) Write down the common difference, d. [1]

(d) Find the amount of fuel pumped into the tank in the $13\mathrm{th}$ hour. [2]

(e) (i) Find the value of n such that $u_n = 0$.

(ii) Write down the number of hours that the pump was pumping fuel into the tank. [3]

At the end of the $2\mathrm{nd}$ hour, the total volume of fuel in the tank was $88\,200\ \mathrm{m}^3$.

(f) Find the total amount of fuel pumped into the tank in the first 8 hours. [2]

(g) Show that the tank will never be completely filled using this pump. [3]

6. [Maximum mark: 17]

Emlyn plays many games of basketball for his school team. The number of minutes he plays in each game follows a normal distribution with mean m minutes.

In any game there is a 30% chance he will play less than 13.6 minutes.

(a) Sketch a diagram to represent this information. [2]

In any game there is a 70% chance he will play less than 17.8 minutes.

(b) Show that $m = 15.7$. [2]

The standard deviation of the number of minutes Emlyn plays in any game is 4.

(c) Find the probability that Emlyn

(i) plays between 13 minutes and 18 minutes in a game;

(ii) plays more than 20 minutes in a game. [4]

There is a 60% chance Emlyn plays less than x minutes in a game.

(d) Find the value of x. [2]

Emlyn will play in two basketball games today.

(e) Find the probability he plays between 13 minutes and 18 minutes in one game and more than 20 minutes in the other game. [3]

Emlyn and his teammate Johan each practise shooting the basketball multiple times from a point X. A record of their performance over the weekend is shown in the table below.

	Emlyn	**Johan**
Saturday	42 successful shots from 70 attempts	16 successful shots from 30 attempts
Sunday	27 successful shots from 32 attempts	51 successful shots from 68 attempts

On Monday, Emlyn and Johan will practise and each will shoot 200 times from point X.

(f) Find the expected number of successful shots Emlyn will make on Monday, based on the results from Saturday and Sunday. [2]

Emlyn claims the results from Saturday and Sunday show that his expected number of successful shots will be more than Johan's.

(g) Determine if Emlyn's claim is correct. Justify your reasoning. [2]